NAG Toolbox for MATLAB

g01gd

1 Purpose

g01gd returns the probability associated with the lower tail of the noncentral **f** or variance-ratio distribution, via the function name.

2 Syntax

```
[result, ifail] = g01gd(f, df1, df2, rlamda, tol, 'maxit', maxit)
```

3 Description

The lower tail probability of the noncentral **f**-distribution with ν_1 and ν_2 degrees of freedom and noncentrality parameter λ, $P(\mathbf{f} \leq f : \nu_1, \nu_2; \lambda)$, is defined by

$$P(F \leq f : \nu_1, \nu_2; \lambda) = \int_0^x p(F : \nu_1, \nu_2; \lambda)\, dF,$$

where

$$P(F : \nu_1, \nu_2; \lambda) = \sum_{j=0}^{\infty} e^{-\lambda/2} \frac{(\lambda/2)^j}{j!} \times \frac{(\nu_1 + 2j)^{(\nu_1+2j)/2} \nu_2^{\nu_2/2}}{B((\nu_1 + 2j)/2, \nu_2/2)}$$

$$\times u^{(\nu_1+2j-2)/2} [\nu_2 + (\nu_1 + 2j)u]^{-(\nu_1+2j+\nu_2)/2}$$

and $B(\cdot, \cdot)$ is the beta function.

The probability is computed by means of a transformation to a noncentral beta distribution:

$$P(F \leq f : \nu_1, \nu_2; \lambda) = P_\beta(X \leq x : a, b; \lambda),$$

where $x = \dfrac{\nu_1 f}{\nu_1 f + \nu_2}$ and $P_\beta(X \leq x : a, b; \lambda)$ is the lower tail probability integral of the noncentral beta distribution with parameters a, b, and λ.

If ν_2 is very large, greater than 10^6, then a χ^2 approximation is used.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **f – double scalar**

f, the deviate from the noncentral **f**-distribution.

Constraint: $\mathbf{f} > 0$.

2: **df1 – double scalar**

The degrees of freedom of the numerator variance, ν_1.

Constraint: $0.0 < \mathbf{df1} \leq 10^6$.

3: **df2 – double scalar**

The degrees of freedom of the denominator variance, ν_2.

Constraint: **df2** > 0.0.

4: **rlamda – double scalar**

λ, the noncentrality parameter.

Constraint: $0.0 \leq$ **rlamda** $\leq -2.0\log(U)$ where U is the safe range parameter as defined by x02am.

5: **tol – double scalar**

The relative accuracy required by you in the results. If g01gd is entered with **tol** greater than or equal to 1.0 or less than $10 \times$ ***machine precision*** (see x02aj), then the value of $10 \times$ ***machine precision*** is used instead.

5.2 Optional Input Parameters

1: **maxit – int32 scalar**

The maximum number of iterations to be used.

Suggested value: 500. See g01gc and g01ge for further details.

Default: 500

Constraint: **maxit** ≥ 1.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g01gd may return useful information for one or more of the following detected errors or warnings.

If on exit **ifail** $= 1$ or 3, then g01gd returns 0.0.

ifail $= 1$

On entry, **df1** ≤ 0.0,
or **df1** $> 10^6$,
or **df2** ≤ 0.0,
or **f** ≤ 0.0,
or **rlamda** < 0.0,
or **maxit** < 1,
or **rlamda** $> -2.0\log(U)$, where $U =$ safe range parameter as defined by x02am.

ifail $= 2$

The solution has failed to converge in **maxit** iterations. You should try a larger value of **maxit** or **tol**.

ifail = 3

The required probability cannot be computed accurately. This may happen if the result would be very close to 0.0 or 1.0. Alternatively the values of **df1** and **f** may be too large. In the latter case you could try using a normal approximation; see Abramowitz and Stegun 1972.

ifail = 4

The required accuracy was not achieved when calculating the initial value of the central **f** (or χ^2) probability. You should try a larger value of **tol**. If the χ^2 approximation is being used then g01gd returns zero otherwise the value returned should be an approximation to the correct value.

7 Accuracy

The relative accuracy should be as specified by **tol**. For further details see g01ge and g01gc.

8 Further Comments

When both ν_1 and ν_2 are large a Normal approximation may be used and when only ν_1 is large a χ^2 approximation may be used. In both cases λ is required to be of the same order as ν_1. See Abramowitz and Stegun 1972 for further details.

9 Example

```
f = 5.5;
df1 = 1.5;
df2 = 25.5;
rlamda = 3;
tol = 5e-06;
[result, ifail] = g01gd(f, df1, df2, rlamda, tol)
```

```
result =
    0.8214
ifail =
           0
```